2000

MATHEMATICS

Full Marks - 1 00
Pass Marks - 33

Time : Three Hours

The figures in the right margin indicate full marks for the corresponding question
For Question Nos. 32-35, write the letter associated with the correct answer

1. Form the differential equation of the family of curves represented by $y^2=4ax$, by eliminating a. 2

2. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$, find AB and BA 2

3. Prove by vector method that the diagonals of a parallelogram bisect each other. 2

4. A balloon released from the ground, rises vertically upwards with an acceleration of 1.5 m/sec^2. A ball is released from the balloon 15 secs after the balloon has left the ground. Find the velocity of the ball at the time of its release frome the balloon. 2

5. In which octants do the following points, viz.
A (2,-2, 3), B (-4, 2,-1), C (-2, -3. 4), D (-4, -3, -2,) lie? 2

6. Prove that $\int_o^a f(x)\,dx = \int_o^a f(a-x)\,dx$. 3

7. Give the definitions of the three types of solution of a differential equation. 3

8. Prove that $(\vec{a} \times \vec{b})^2 = a^2 + b^2 - (\vec{a}.\vec{b})^2$ 3

9. Prove that if two rows (columns) of a determinant are identical, then its

value is zero.
3

10. Find from first principles the derivative of e^x with respect to x. 3

11. Find the value of $\int \frac{x}{((x-1)(x-2)} dx$ 3

12. Prove that

$\frac{\pi}{2}$ √ cost x

π

$$\int \frac{\quad}{a \sqrt{\sin} + \sqrt{\cos x}} dx = \frac{\quad}{2}$$

13. Solve (x+y+l) dy/dx=1. 3

14. Find the unit vector perpendicular to the plane of the vectors.

--> A A A --> A A A
a = l + j + k and b = l - 2k 3

15. Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & C^2 \end{vmatrix}$ = (a-b) (b-c) (c-a). 3

16. Using matrix method, solve

4x-3y=5

3x-5y= 1.

3

17. Draw the graph of *f* : x .⟶ | x | + | x+1|, X=R

3

18. Prove that $\int e^{ax} \sin (bx+c) =dx = \frac{e^{ax} \sin(bx+c-\tan^{-1} b/a)}{\sqrt{a^2 + b^2}} + C$ 3

5

19. A particle moves along a straight line with initial velocity u and constant celeration a in its direction of motion. *If* v be its final velocity at the end of the time t and s be the distance moved during this time t, prove that

i) v=u + at,
ii) s=ut+ l/2 at^2,
iii) $V^2=U^2+2as$. 1 +2+2=5

20, Change the equation of the line .

$a_1x+b_1y+c_1z+d_1=o=a_2x+b_2y+c_2z+d_2$

into its symmetrical form 5

21. Test the continuity of

$f(x)= \frac{x^3-2x^2-x+2}{x-2}$ for x02

=0 for x=2

at the point x=2. 5

22. Prove that. $\int_0^{\frac{\pi}{2}} \text{In} \cos x \, dx = \frac{\pi}{2}$ in 1/2

23. Two equal forces each of magnitude P acting at a point has a resultant of magnitude P. *If one* of the forces is doubled, prove that the new resultant makes a right angle with the other force. 5

24. Find the equation of the sphere passing through the points 0 (o,o,o), A (a,o,o), B (o,b,o) and C (o,o,c). Also find its centre and the radius. 3+1+1=5

25. Water is being pumped into a conical reservoir whose height is 4m and radius of its top is 2m at a constant rate of 11/ 7 m^3/min. How fast is the water level rising when it is 2m deep ? 5

26. If t be the time in which a projected particle reaches a point P of its path, and t' be time from P till it strikes the horizontal plane through the point of projection, show that the height of P above the plane is 1/2gtt'. 5

27. Give the definition of continuity of a function at a point x=a. 1

28. Give the *two* general conditions for the equilibrium of *three forces.* 1

29. What is the value of dy/dx when, $y = (I + X^2)^7$ 1

30. What is the value of the resolved part of any force F in a direction at right angles to itself ? 1

31. Is a matrix a number?

32. For XE R, $\lim_{X \to oc} (1 + 1/X)^X$ is equal t o

(A) 0 (B) 1
(C) e (D) oc.

33. The value of d/dx (sec^{-1} x) is

A. $\dfrac{1}{x\sqrt{x^2-1}}$

B. $\dfrac{1}{x^2\sqrt{x-1}}$

C. $\dfrac{1}{x\sqrt{1-x^2}}$

D. $\dfrac{1}{x^2\sqrt{1-x}}$ 1

34. The area of the region bounded by the curve **$y=X^2$,** x-axis and the straight lines x=2 and x=5 in square units is

(A) 36 (B) 3 7 (C) 38 (D) 39. 1

35. Two forces P and Q and their resultant R act at a point

0. If their directions meet a transversal in L, M, N respectively, then : 1

(A) $\dfrac{P}{OL} + \dfrac{Q}{OM} + \dfrac{R}{ON} = 0$

(B) $\dfrac{P}{OL} + \dfrac{Q}{OM} - \dfrac{R}{ON} = 0$

(C) $\dfrac{P}{OL} - \dfrac{Q}{OM} + \dfrac{R}{ON} = 0$

(D) $\dfrac{P}{OL} - \dfrac{Q}{OM} - \dfrac{R}{ON} = 0$

MATHEMATICS

Scoring Key /Outline Answer And
Marking Scheme,

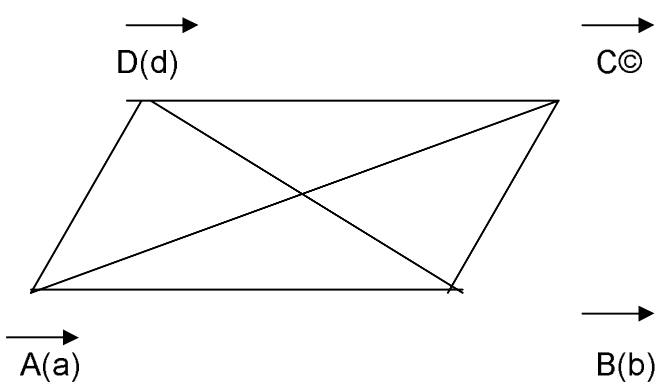
1. **H**ere, $Y^2=4ax$

Diff. w.r.t. x, **2y dY/dx** =4a-= $\mathbf{Y^2/x}$

= y dx – 2x dy = 0

2. $AB = \begin{bmatrix} 5 & 0 \\ 6 & 8 \end{bmatrix}$

$BA = \begin{bmatrix} 11 & --6 \\ 3 & 2 \end{bmatrix}$



Let a, b, c, d, be t he Position vectors of the vertices A, B, C, D of the pars. ABCD.

Then $\overrightarrow{AB} = \overrightarrow{DC}$

$= \vec{b} - \vec{a} = \vec{c} - \vec{d}$

$= \vec{b} + \vec{d} = \vec{a} + \vec{c}$

$= \frac{1}{2} (\vec{b} + \vec{d}) = \frac{1}{2} (\vec{a} + \vec{c})$

P.V of the mid pt. Of BD is the same as the P.V of the mid pt. Of $\overrightarrow{AC}$

Hence the result

4. The velocity of the ballon after 15 secs. = 0 +1.5 x 15 = 22.5 m/sec 1

The velocity of the ball at the time of release is equal to the velocity of the ballon at that moment.

∴ Velocity of the ball is 22.5 m/sec 1

5. A. iii : oxy'z

B. vi : ox'yz'

C. v : ox'y'z

D. viii : ox'y'z'

6. Let $\vec{x} = a - y$, then dx = - dy, when x = a, y = 0

and if x = 0, y = a

$$\therefore \int_o^{\alpha} f(x)\, dx = -\int_a^{o} f(a-y)\, dy$$

$$\int_o^{\alpha} f(a-y)\, dy = \int_o^{\alpha} f(a-x)\, dx \qquad 1$$

7. Defns. of (i) General solution 1

(ii) Particular solution 1

(iii) Singular solution 1

8. L.S.= $(\vec{a} \times \vec{b})$ = $(|\vec{a}| \; |\vec{b}| \; \text{Sin } \theta \hat{\eta}).2$ Where θ is the

angle between a and b. 1

$= |\vec{a}|^2 \; |\vec{b}|^2 \sin^2\theta$ 1/2

$= |\vec{a}|^2 \; |\vec{b}|^2 (1 - \text{Cos}^2\theta)$ 1/2

$= |\vec{a}|^2 \; |\vec{b}|^2 - a^2b^2 \text{Cos}^2\theta$ 1/2

$= |\vec{a}|^2 \; |\vec{b}|^2 - (\vec{a}.\vec{b})^2$ ½

= R. S.

9. Consider a det. △ having two rows identical. Interchange the two identical rows and denote it by △

$\therefore \triangle = \triangle$ ………….. (i)
and $\triangle' = - \triangle$ (by prop. 3) ………… (ii)
(i) and (ii) $\rightarrow \triangle = - \triangle \rightarrow 2 \triangle = 0$
$\rightarrow \triangle = 0$ 1

10. $\frac{d}{dx} e^x = \lim_{\delta x \to 0} \frac{e^{x+\delta x} - e^x}{x}$

$= \lim_{\delta x \to 0} \frac{ex\,(e\,\delta x - 1)}{\delta x}$

$= e^X$

11. Choosing of proper method (partial fraction). Integration process i.e. 1

x 2 1

$$\int \frac{\quad}{(x-1)(x-2)}\,dx = \int \left(\frac{\quad}{x-2} - \frac{\quad}{x-1}\right) dx$$

$$= \ln\left|\frac{(x-2)^2}{(x-1)}\right| + c$$

12. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$ 1

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}\ \sqrt{\cos x}}\,dx$$

$$\int^{\frac{\pi}{1}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

$$\therefore\ I + I == \int_0^{\frac{\pi}{1}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} dx = [\,x\,]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$I = \frac{\pi}{4}$ 1

13. Here, $(x + y + 1) \frac{dy}{dx} = 1$ 1/2

$= \frac{dy}{dx} = x + y + 1$

$= \frac{dx}{dy}$ **-- x = y + 1**

Now, I.F = $e^{\int - dy} = \mathbf{e^{-y}}$ ½

$\therefore\ x.e^{-y} = \int (y + 1)\, e^{-y}\, dy + c$

$= -- (y + 1)\, e^{-y} -- e^{-y} + c$

$= x = --(y + 1)^{-1} + c\, e^{y}$

$=x =--(y +2)+c\, e^{y}$

Which is the reqd. solution,

14. Take $\vec{c}$ perp. to both $\vec{a}$ and $\vec{b}$, then,

$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{I} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & --2 \end{vmatrix}$

$$= 2\hat{i} + 3\hat{j} - \hat{k}$$

$\therefore |c| = \pm\sqrt{14}$ 1/2

$\therefore$ The unit vector perp. to a and b 1/2

$= 1/\pm\sqrt{14}\,(2\hat{i} - 3\hat{j} + \hat{k})$ 1/2

15. L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-c^2 & c^2-a^2 \end{vmatrix}$

By $c_2 - c_1$, $c_3 - c_1$ 1

$= (b-a)(c-a)\begin{vmatrix} 1 & o & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$

$= (b-a)(c-a)\{(c+a)-(b+a)\}$ ½

$= (b-a)(c-a)(c-b)$ ½

$= (a-b)(b-c)(c-a)$

= R.H.S. 1

16. $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

= AX = B ½

$|A| = -11 \neq 0$ ½

Adj A $= \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \Rightarrow A^{-1} = -1/11\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$ ½ + ½

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5/11 & -3/11 \\ 3/11 & -4/11 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\therefore$ x=2, y=1 1

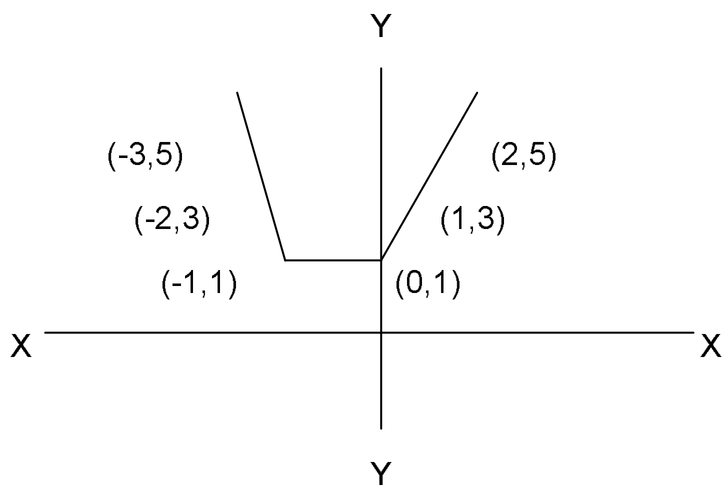
17.

x	-3	-2	-1	0	1	2
f (x)	5	3	1	1	3	5

1

Taking axes & plotting 1

Graph 1



18. Book article 7.41 : Integration by parts 1

Transporting and simplification 2

Proper substitution 8 Cosec = a etc. 1

Simplification to get R.H.S. 1

19. Book article 9.6 (i) v = u+ft 1

(ii) $s=ut+1/2at^2$ 2

(iii) $v^2=u^2+2as$ 2

20. Book article 3.4.3 : To find d.r's 2

To find a point on the line 2

Writing the symmetrical from 1

21. $\lim_{x \to 2} f(x) = 3$ (Finding L.H. Lim and R.H. Lim) ½+1/27

F (2)=0

$\therefore \lim_{y \to 2} f(x) \neq f(2)$

Hence, f(x) is discontinuous at x=2 1

22.

$$I = \int_0^{\pi/2} \ln \cos x \, dx = \int_0^{\pi/2} \ln \cos(\pi/2 - x)\, dx$$

$$= \int_0^{\pi/2} \ln \sin x \, dx$$

$$\therefore 2I = \int_0^{\pi/2} \ln \sin 2x/2 \, dx = -\pi/2 \log 2 + \int_0^{\pi/2} \ln \sin 2x dx \quad 1\tfrac{1}{2}$$

$= -\pi/2 \log 2 + I$

$= I = \pi/2 \log ½$

H. P.

23. To find the angle betwean the two equal *forces,* *2*

$\theta = 120^\circ$

$$\text{Again, } \tan\alpha = \frac{2p\sin\theta}{1+2p\cos\theta} = \infty$$

$= \alpha = \pi/2$ (α being the direction of the resultant) 1

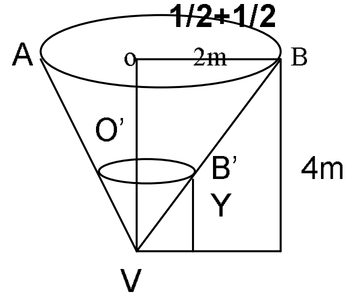
24. To find equation of the sphere is $x^2+y^2+z^2$-ax-by

$- cz = 0$ 3

Centre is (a/2, b/2, c/2) 1

Radius is $\frac{1}{2}\sqrt{a^2+b^2+c^2}$ 1

25. Let v be the volume of the water in the reservoir at time t. Let x be the radius of the section of the cone at the water level and y be the depth of water in the reservoir at time t, **1/2+1/2**



By Qs. $dv/dt = 11/7$ …….(i)

$V = \pi/3 x^2 y$ ……..(ii)

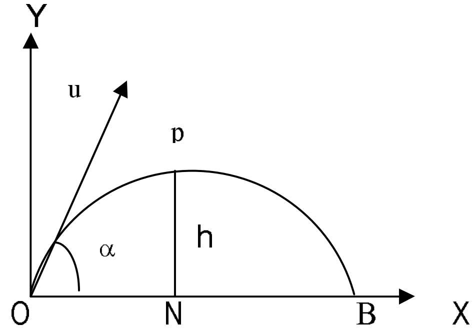
From the fig $x/y = 2/4 = \frac{1}{2} \rightarrow x = \frac{1}{2} y$

i.e, $v = \pi/12\, y^3$

$\Rightarrow dv/dt = \pi/12\, 3y^2\, dy/dt = \pi/4\, y^2\, dy/dt$

$\therefore \left(\frac{dy}{dt}\right) = \frac{4 \times 11/7}{\pi \times 4} = 0.5$ m / min

t = 2 1



26. Let α be the angle of projection and u be the velocity of projection.

Then, $o = (u \sin\alpha).(t+t') - \frac{1}{2} g (t+t')^2$

$\Rightarrow u \sin\alpha = \frac{1}{2} g (t+t')$………(i)

If h be the height of p above the - plane,

Then, $h = (u \sin\alpha).T - \frac{1}{2} gt^2$

$= \frac{1}{2} g (t+t') t - \frac{1}{2} gt^2$, by (i)

$\Rightarrow h = \frac{1}{2} gtt^2$ H. P.

27. Book defn. $\lim_{x \to a} f(x) = f(a)$

28. (i) When the three forces are coplanar. 1/2

(ii) When the three forces are concurrent or parallel. 1/2

29.	$14x (I+X^2)^6$	
30,	0	1
31.	No	1
32.	C	1
33.	A	1
34.	D	1
35.	B	1

N. B. : For question Nos. 8 and 12, full marks should be given to the candidates who have attempted.

HEAD EXA MINER
MATHEMATICS